

Math 10560 Exam 2**Oct. 13, 2026.**

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off (and Put Away) all cellphones, smartwatches and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 19 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-16.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1 ☐	(•)	(b)	(c)	(d)	(e)
2 ☐	(•)	(b)	(c)	(d)	(e)
.....					
3.	(•)	(b)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
.....					
5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
.....					
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
.....					
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)
.....					
11.	(•)	(b)	(c)	(d)	(e)
12.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.**Multiple Choice** _____

13. _____

14. _____

15. _____

16. _____

Total _____

Name (IN CAPITALS): _____

Instructor: _____

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2	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice _____

13. _____

14. _____

15. _____

16. _____

Total _____

Multiple Choice

1.(7pts) Which of the following has the same value as the definite integral

$$\int_1^2 \frac{3x^2 - x + 12}{x(x^2 + 4)} dx.$$

Solution. Use partial fractions with constants A , B , and C :

$$\frac{3x^2 - x + 12}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}.$$

Clearing denominators gives

$$3x^2 - x + 12 = A(x^2 + 4) + x(Bx + C) = (A + B)x^2 + Cx + 4A.$$

Matching coefficients, $4A = 12$, $C = -1$, and $A + B = 3$, so $A = 3$, $B = 0$, and $C = -1$.
Hence

$$\frac{3x^2 - x + 12}{x(x^2 + 4)} = \frac{3}{x} - \frac{1}{x^2 + 4}.$$

(a) $\int_1^2 \frac{3}{x} dx - \int_1^2 \frac{1}{x^2 + 4} dx.$

(b) $\int_1^2 \frac{1}{x} dx + \int_1^2 \frac{1}{x^2 + 4} dx.$

(c) $\int_1^2 \frac{2}{x} dx + \int_1^2 \frac{x}{x^2 + 4} dx.$

(d) $\int_1^2 \frac{2}{x} dx + \int_1^2 \frac{x - 1}{x^2 + 4} dx.$

(e) $\int_1^2 \frac{3}{x} dx - \int_1^2 \frac{x - 1}{x^2 + 4} dx.$

3.

Initials: _____

2.(7pts) Use a rationalizing substitution to calculate the integral

$$\int_1^4 \frac{dx}{x + \sqrt{x}}.$$

Solution. Set $t = \sqrt{x}$, so $x = t^2$ and $dx = 2t \, dt$. The bounds become $t = 1$ and $t = 2$. Then

$$\int_1^4 \frac{dx}{x + \sqrt{x}} = \int_1^2 \frac{2t}{t^2 + t} \, dt = 2 \int_1^2 \frac{dt}{t + 1} = 2 [\ln(t + 1)]_1^2 = 2 (\ln 3 - \ln 2).$$

(a) $2 (\ln(3) - \ln(2))$

(b) $\ln(5) - \ln(2)$

(c) $2 (\ln(5) - \ln(2))$

(d) $2 (\tan^{-1}(3) - \tan^{-1}(1))$

(e) $\frac{1}{2} (\ln(3) - \ln(2))$

4.

Initials: _____

3.(7pts) Use Simpson's rule with $n = 4$ to estimate

$$\int_1^5 \frac{1}{x^2} dx.$$

Solution. For $n = 4$, the step size is $h = (5 - 1)/4 = 1$. The nodes are $x_i = 1, 2, 3, 4, 5$, and for $f(x) = 1/x^2$, the values are $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}$. Composite Simpsons rule gives

$$S_4 = \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)] = \frac{1}{3} \left[1 + 1 + \frac{2}{9} + \frac{1}{4} + \frac{1}{25} \right]$$

(a) $\frac{1}{3} \left[1 + 1 + \frac{2}{9} + \frac{1}{4} + \frac{1}{25} \right]$

(b) $\frac{1}{2} \left[1 + \frac{1}{2} + \frac{2}{9} + \frac{1}{8} + \frac{1}{25} \right]$

(c) $\frac{1}{3} \left[1 + \frac{1}{2} + \frac{2}{9} + \frac{1}{8} + \frac{1}{25} \right]$

(d) $\frac{1}{2} \left[1 + 1 + \frac{4}{9} + \frac{1}{4} + \frac{1}{25} \right]$

(e) $\frac{1}{2} \left[1 + \frac{1}{2} + \frac{1}{9} + \frac{1}{8} + \frac{1}{25} \right]$

5.

Initials: _____

4.(7pts) If the trapezoidal rule with n approximating intervals is used to estimate

$$\int_1^3 \frac{1}{x^3} dx.$$

Which of the values of n given below is the least value of n for which we can be sure that the error of approximation is at most $1/2$ using the error bound

$$|E_T| \leq \frac{K(b-a)^3}{12n^2},$$

where $|f''(x)| \leq K$ on $[a, b]$.

Solution. For $f(x) = x^{-3}$, compute the second derivative:

$$f''(x) = 12x^{-5} = \frac{12}{x^5}.$$

On $[1, 3]$, this is largest in absolute value at $x = 1$, so we can take $K = 12$. With $a = 1$ and $b = 3$, the error bound becomes

$$|E_T| \leq \frac{12(3-1)^3}{12n^2} = \frac{8}{n^2}.$$

Require this bound to be at most $\frac{1}{2}$:

$$\frac{8}{n^2} \leq \frac{1}{2} \implies n^2 \geq 16 \implies n \geq 4.$$

Thus the least suitable value is $n = 4$.

- (a) $n = 4$ (b) $n = 2$ (c) $n = 6$ (d) $n = 8$ (e) $n = 10$

6.

Initials: _____

5.(7pts) Consider the three integrals

$$\text{I. } \int_{-1}^1 \frac{dx}{x^3} . \quad \text{II. } \int_{-1}^1 \frac{dx}{\sqrt[3]{x}} . \quad \text{III. } \int_0^1 \frac{dx}{x-1} .$$

One of the following statements is true. Which one?

Solution. The p -integral $\int_0^1 \frac{dx}{x^p}$ converges only if $p < 1$. Values of p for integrals I, II and III are 3, $1/3$ and 1, respectively. Therefore, the only convergent integral above is II.

- (a) II is convergent; I and III are divergent.
- (b) They are all divergent.
- (c) They are all convergent.
- (d) I is convergent; II and III are divergent.
- (e) II and III are convergent; I is divergent.

7.

Initials: _____

6.(7pts) Consider the improper integral

$$\int_{-1}^1 \frac{dx}{3x-1}.$$

One of the following statements is true. Which one?

Solution. The integral is divergent by the same reasoning as in Question 5. Take $p = 1$.

(a) The integral is divergent.

(b) The integral converges to $-\frac{\ln(2)}{3}$.

(c) The integral converges to $\frac{1}{3}$.

(d) The integral converges to 0.

(e) The integral converges to $-\ln(2)$.

8.

Initials: _____

7.(7pts) Evaluate the following integral $\int_2^{+\infty} x e^{-x^2} dx$.

Solution. Let $u = x^2$, so $du = 2x dx$. Treating the improper integral as a limit gives

$$\begin{aligned}\int_2^{\infty} x e^{-x^2} dx &= \lim_{b \rightarrow \infty} \int_2^b x e^{-x^2} dx \\ &= \lim_{b \rightarrow \infty} \left[-\frac{1}{2} e^{-x^2} \right]_2^b = \frac{1}{2e^4}.\end{aligned}$$

(a) $\frac{1}{2e^4}$

(b) $\frac{e^2}{2}$

(c) e^2

(d) $-e^2$

(e) $-\frac{1}{e^4}$

8.(7pts) Which of the following is the correct expression for the arc length of the curve

$$y = \sin(3x)$$

from $x = 0$ to $x = \frac{\pi}{2}$?

Solution. The arc length of $y = f(x)$ on $[a, b]$ is $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$. Here $f'(x) = 3 \cos(3x)$. Substituting this derivative and the given bounds gives

$$L = \int_0^{\pi/2} \sqrt{1 + 9 \cos^2(3x)} dx.$$

(a) $\int_0^{\pi/2} \sqrt{1 + 9 \cos^2(3x)} dx$

(b) $\int_0^{\pi/2} \sqrt{1 + \sin^2(3x)} dx$

(c) $\int_0^{\pi/2} \sqrt{1 - 9 \cos^2(3x)} dx$

(d) $\int_0^{\pi/2} \sqrt{1 - \sin^2(3x)} dx$

(e) $\int_0^{\pi/2} \sqrt{1 + \cos^2(3x)} dx$

10.

Initials: _____

9.(7pts) Compute the distance travelled by a particle moving along the curve $y = \ln(|\sec x|)$ from the point $(0, 0)$ to the point $(\pi/4, \ln(\sqrt{2}))$

Note: The formula sheet may help with this problem.

Solution. On $[0, \pi/4]$, $\sec x > 0$, so the curve is $y = \ln(\sec x)$. Its derivative is $dy/dx = \tan x$. Using the arc-length formula,

$$L = \int_0^{\pi/4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx = \int_0^{\pi/4} \sec x dx.$$

Since an antiderivative of $\sec x$ is $\ln|\sec x + \tan x|$,

$$L = [\ln|\sec x + \tan x|]_0^{\pi/4} = \ln(\sqrt{2} + 1).$$

- (a) $\ln(\sqrt{2} + 1)$ (b) $\frac{1 - \sqrt{2}}{2}$ (c) $\frac{\sqrt{2} - 1}{2}$ (d) $\ln(3)$ (e) $\frac{\pi}{8} - \frac{1}{4}$

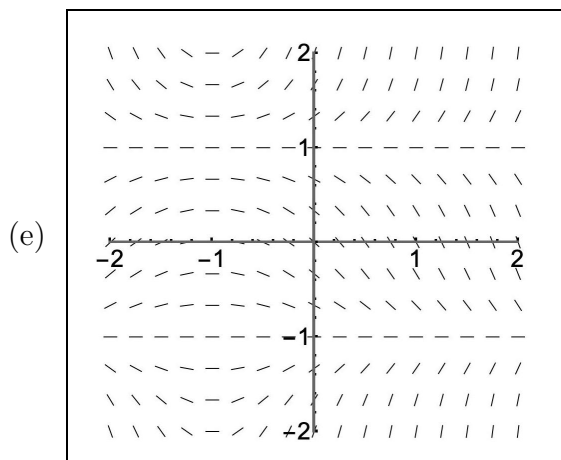
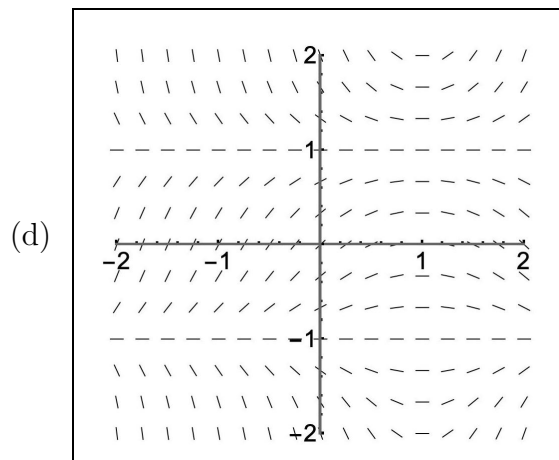
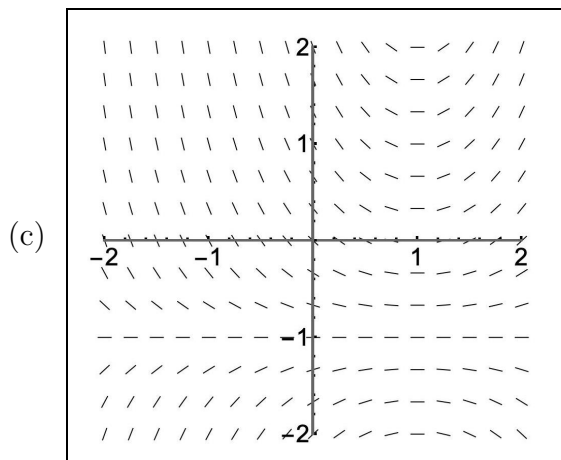
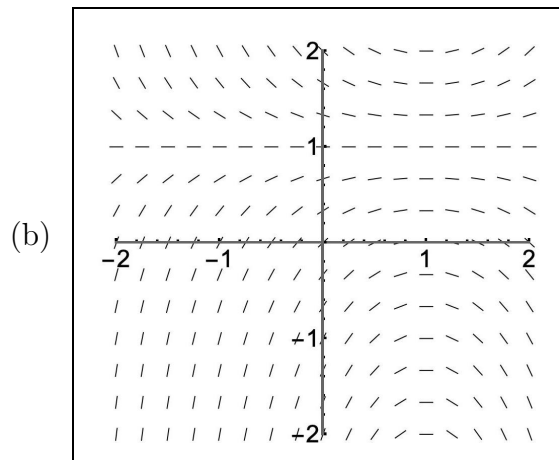
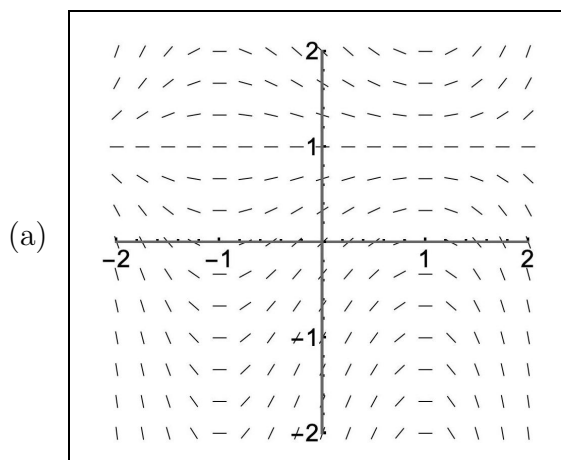
11.

Initials: _____

10.(7pts) Which of the following gives the direction field for the differential equation

$$\frac{dy}{dx} = (y - 1)(x^2 - 1) ?$$

Solution. Can be easily identified by noticing $dy/dx = 0$ when $x = \pm 1$ or $y = 1$, which means that the slope should be horizontal on the horizontal line $y = 1$ and on vertical lines $x = \pm 1$.



12.

Initials: _____

- 11.(7pts) Use Euler's method with step size 0.5 to estimate $y(1)$ where $y(x)$ is the solution to the initial value problem

$$y' = x + y - 1, \quad y(0) = 1.$$

Solution. The recursion of Euler's method is $y_{n+1} = y_n + hf(x_n, y_n)$, where $f(x, y) = x + y - 1$ and $h = 0.5$. Starting from $(x_0, y_0) = (0, 1)$,

$$f(0, 1) = 0, \quad y_1 = 1 + 0.5(0) = 1,$$

so the next point is $(x_1, y_1) = (0.5, 1)$. The next slope is $f(0.5, 1) = 0.5$, giving

$$y_2 = 1 + 0.5(0.5) = 1.25 = \frac{5}{4}.$$

Thus the estimate for $y(1)$ is $\frac{5}{4}$.

(a) $y(1) \approx \frac{5}{4}$

(b) $y(1) \approx \frac{3}{2}$

(c) $y(1) \approx \frac{1}{2}$

(d) $y(1) \approx \frac{1}{4}$

(e) $y(1) \approx \frac{3}{4}$

13.

Initials: _____

12.(7pts) Find the solution of the initial value problem

$$\frac{dy}{dx} = y^4 \sin(x), \quad y(0) = 1.$$

Solution. Separate variables:

$$y^{-4} dy = \sin x \, dx.$$

Integrating both sides gives

$$-\frac{1}{3y^3} = -\cos x + C.$$

Apply $y(0) = 1$: $-\frac{1}{3} = -1 + C$, so $C = \frac{2}{3}$. Rearranging,

$$\frac{1}{y^3} = 3 \cos x - 2, \quad y = \left(\frac{1}{3 \cos x - 2} \right)^{1/3}.$$

(a) $y = \left(\frac{1}{3 \cos x - 2} \right)^{1/3}$

(b) $y = \left(\frac{4}{\cos x + 3} \right)^{1/5}$

(c) $y = (6 - 5 \cos x)^{1/5}$

(d) $y = \left(\frac{5 - \cos x}{4} \right)^{1/3}$

(e) $y = \left(\frac{1}{4 - 3 \cos x} \right)^{1/3}$

Partial Credit

For full credit on partial credit problems, make sure you justify your answers.

13.(14pts) Compute

$$\int \frac{2}{(x-1)(x^2+1)} dx$$

Solution. Write the integrand using partial fractions:

$$\frac{2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}.$$

Multiplying by $(x-1)(x^2+1)$ gives

$$2 = A(x^2+1) + (Bx+C)(x-1) = (A+B)x^2 + (C-B)x + (A-C).$$

Equating coefficients yields $A+B=0$, $C-B=0$, and $A-C=2$. Solving gives $A=1$, $B=-1$, and $C=-1$. Therefore,

$$\frac{2}{(x-1)(x^2+1)} = \frac{1}{x-1} - \frac{x+1}{x^2+1}.$$

Integrate each term:

$$\begin{aligned} \int \frac{2}{(x-1)(x^2+1)} dx &= \int \frac{1}{x-1} dx - \int \frac{x}{x^2+1} dx - \int \frac{1}{x^2+1} dx \\ &= \boxed{\ln|x-1| - \frac{1}{2}\ln(x^2+1) - \arctan x + C} \end{aligned}$$

where C is an arbitrary constant.

15.

Initials: _____

14.(11pts) Find the orthogonal trajectories of the family of curves

$$y = \frac{K}{\sqrt{x}}$$

Solution. The given family is $y = Kx^{-1/2}$, with K constant. Differentiating with respect to x gives

$$\frac{dy}{dx} = -\frac{K}{2}x^{-3/2} = -\frac{y}{2x}.$$

At each intersection, orthogonal curves have perpendicular tangent lines, so their slopes are negative reciprocals. Thus, the orthogonal trajectories satisfy

$$\frac{dy}{dx} = \frac{2x}{y}.$$

Separate variables and integrate:

$$y \, dy = 2x \, dx \quad \implies \quad \frac{y^2}{2} = x^2 + C.$$

Therefore, the family of orthogonal trajectories is

$$\boxed{y^2 - 2x^2 = C}$$

where C is an arbitrary constant.

16.

Initials: _____

True-False.

15.(3pts) Please circle “TRUE” if you think the statement is true, and circle “FALSE” if you think the statement is False.

(a)(1 pt. No Partial credit) Simpson’s rule can be used to find the exact value of any definite integral.

TRUE

FALSE

(b)(1 pt. No Partial credit)

$\int_1^\infty \frac{1}{\sqrt{x}} dx$ diverges.

TRUE

FALSE

(c)(1 pt. No Partial credit) The slope field for the differential equation $\frac{dy}{dx} = ye^{x^2}$ has a line with slope -1 at the point $(-1, 0)$.

TRUE

FALSE

17.

Initials: _____

16.(2pts) You will be awarded these two points if you write your name in CAPITALS on the front page and you mark your answers on the front page with an X through your answer choice like so: ~~A~~ (not an O around your answer choice) .

The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$

$$\int \sec \theta = \ln |\sec \theta + \tan \theta| + C$$

$$\int \csc \theta = \ln |\csc \theta - \cot \theta| + C$$

$$\csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

19.

Initials: _____

ROUGH WORK